The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus
Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Polynomial Approximations (Exact data Method)

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2nd Approach of Polynomial Approximation is

2- Polynomial Approximations - exact data.

In this approach we consider cases where, rather than knowing an expression for the function, we have a list of point values.

→ Approximating polynomial.

Sometimes it is good enough to find a polynomial that passes near these points i.e like putting a straight line through experimental data. Such a polynomial is an approximating polynomial as shown in fig (a)

⇒ Interpolating polynomial :-

→ For the case of exact data.

⇒ Topic:- polynomial Approximation (Exact data)

(35)

we deal with the case where a polynomial to pass exactly through the given data, that is an interpolating polynomial as shown in Fig (b).

Interpolate → means extend or join → joint —

Lang

⇒ Lagrange Interpolation:-

In lagrange interpolation we have a function 'f' exactly at a few points and our goal to approximate how the function behaves between those (given) points. Or it is often more desirable to seek a curve that does not have corners in it as in Fig (b)

For exp

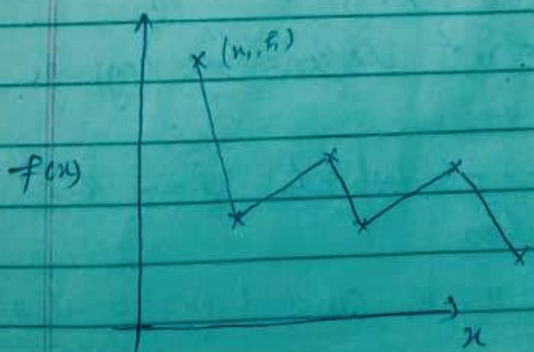

Fig (a)

(a) Linear, or dot-to-do interpolation with corners at all of data points.

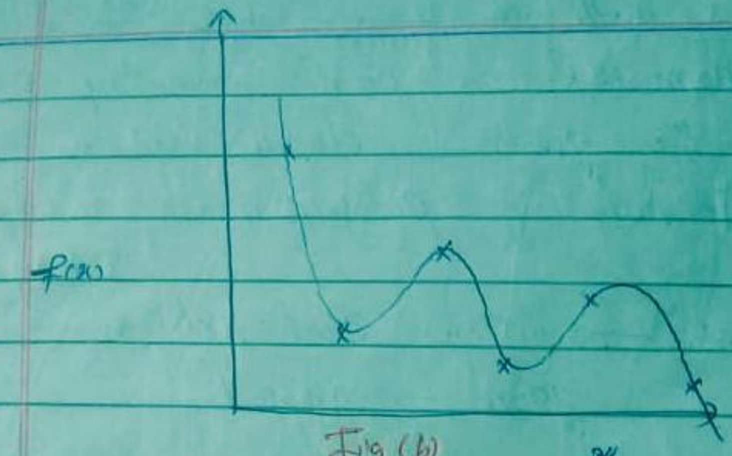

Fig (b)

(b) A smoother interpolation of the data points. (more desirable)

Let our data are in the form
$\{(x_1, f_1), (x_2, f_2), (x_3, f_3), \ldots\}$.
These are the points plotted across on the above diagrams i.e Fig. a & b

→ our aim to find polynomial which exacly passes through the given data.

→ we want to find $p(x)$ such that
$$p(x_1) = f_1, \; p(x_2) = f_2, \; p(x_3) = f_3 \ldots$$
For this we first define the Lagrange polynomial, as
$L_1, L_2, L_3 \ldots$ which have the following properties.

$L_1(x) = 1$, at $x = x_1$ and $L_1(x) = 0$ at $x = x_2, \ldots$

$L_2(x) = 1$, at $x = x_2$ & $L_2(x) = 0$ at $x = x_1, x_3$

$L_3(x) = 1$, at $x = x_3$ & $L_3(x) = 0$ at $x = x_1, x_2, x_4$

Each of these functions acts like a filter which 'turns off' if you evaluate it at the data point other than its own.

(ii) The Lagrange polynomial L_1, L_2, L_3 — which returns 1 for its own point and returns zero for any other point

These two properties are enough to be able to write down what $p(x)$ must be. or we get the goal of polynomial with approximation.

2=c

$$p(x) = f_1 L_1(x) + f_2 L_2(x) + f_3 L_3(x) + \cdots$$

This filtering property of the lagrange polynomial picks out exactly the right f-value for the current x-value.

Between the data points, the expression for 'p' will give a smooth polynomial curve.

For exp $x = x_2$

$$p(x_2) = f_1 \overset{0}{L_1(x_2)} + f_2 \overset{1}{L_2(x_2)} + f_3 \overset{0}{L_3(x_2)} + \cdots$$

$$= f_2 \{1\}$$

$$p(x_2) = f_2$$

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)\ldots}{(x_1-x_2)(x_1-x_3)(x_1-x_4)\ldots}$$

$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)\ldots}{(x_2-x_1)(x_2-x_3)(x_2-x_4)\ldots}$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)\ldots}{(x_3-x_1)(x_3-x_2)(x_3-x_4)\ldots}$$

and so on.

Note:–
The numerator of $L_i(x)$ does not contain $(x-x_i)$
And
The denominator of $L_i(x)$ does not contain (x_i-x_i)

For Exp:–
Use the Lagrange interpolation to estimate "$f(8)$" to appropriate accuracy give the table of ~~[illegible]~~ values below

x	2	5	7	9	10
$f(x)$	0.980067	0.8775836	0.764842	0.621610	0.5403

Example of the Exact Data Method.

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Now $x = 8$; $x_1 = 5$, $x_2 = 7$
$x_3 = 9$, $x_4 = 10$

Now $f_1(x_1) = f_1(5) = 0.8775836$
$f_2(x_2) = f_2(7) = 0.764842$
$f_3 = f_3(9) = 0.621610$
$f_4 = f_4(10) = 0.540302$

As
$$p(x) = f_1 L_1(x) + f_2 L_2(x) + f_3 L_3(x) + f_4 L_4$$

$$p(8) = \frac{(8-7)(8-9)(8-10)}{(5-7)(5-9)(5-10)} \times 0.877583 \qquad \frac{(1)(-1)(-2)}{(-2)(-4)(-5)}$$

$$+ \frac{(8-5)(8-9)(8-10)}{(7-5)(7-9)(7-10)} \times 0.764842 \qquad \frac{(3)(-1)(-2)}{(2)(-2)(-3)}$$

$$+ \frac{(8-5)(8-7)(8-10)}{(9-5)(9-7)(9-10)} \times 0.621610 \qquad \frac{(3)(1)(-2)}{(4)(2)(-1)}$$

$$+ \frac{(8-5)(8-7)(8-9)}{(10-5)(10-7)(10-9)} \times 0.540302 \qquad \frac{(3)(1)(-1)}{(5)(3)(1)}$$

$$p(8) = -\frac{1}{20} \times 0.877583 + \frac{1}{2} \times 0.764842 + \frac{3}{4} \times 0.621610 - \frac{1}{5} \times 0.540302$$

$$p(8) = 0.6966689$$

Suitable accuracy is 0.6967 (Round off to 4 d.p)

Best of Luck